Bayesian confusion[1]

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ABSTRACT

The purportedly bayesian analysis of Niels Lind (2002) contains two flaws. First, Lind has mixed up different hypotheses. Most importantly, he has made an un-bayesian and erraneous estimation of the prior probability, avoiding to analyse the factual circumstances. Lind's conclusion is thus invalid.

INTRODUCTION

Jes Wienberg's (2001) paper on pseudoarchaeology and "sacred topography" deals, among other things, with Erling Haagensen's claims that medieval churches of Bornholm have been deliberately aligned. Four of the 15 churches align pairwise with Christiansø, situated approximately 25 kms north of Bornholm. Wienberg remains sceptical. On the other hand, Niels Lind (2002) has performed a bayesian analysis from which he concludes that "it is almost certain that some of the 15 medieval churches were deliberately aligned with Christiansø" (p. 55). Unfortunately, Lind's analysis is fallacious.

CONFUSION OF HYPOTHESES

First, a technical flaw. Let us follow Lind in letting *a* denote the actual coordinates of the churches, whereas *b* denotes the hypothesis "none of the 15 medieval churches were deliberately aligned with Christiansø", and *c* is the complementary hypothesis "some of the 15 medieval churches were deliberately aligned with Christiansø" (p. 51f). Adding the probabilities of *b* and *c*, the result is one; $P(b) + P(c) = 1$. Lind wishes to estimate $P(b|a)$, the probability of *b* given the actual locations of the churches. In order to use Bayes's theorem $P(a|b)$ must be calculated, i.e. the probability of the actual locations given that *b* is true. Lind states: "If none of the 15 medieval churches were deliberately aligned with Christiansø, then any alignment must be accidental, due to random coincidence of the church-to-Christiansø azimuths" (p. 52). Lind then proceeds by calculating $P(a|b)$ given a model of randomly located churches.

But that is a questionable step. There are three cases: (1) Churches were deliberately aligned with respect to Christiansø. (2) Churches were deliberately aligned, but not with respect to Christiansø. (3) Churches were not deliberately aligned. (1), (2) and (3) cover all possibilities. Only one of them is actually true. Lind equates the hypothesis *c* with (1). According to Lind's original definition, the *b* hypothesis corresponds to the complement of (1), i.e. the union of (2) and (3), but later on Lind represents *b* by a chance model, which might refer to (3), but hardly to (2). Discussing the choice of prior probability, Lind writes: "[A] person reasonably skeptical about Haagensen's theory could a priori assign probabilities P(b0) = 0.95 and therefore P(c0) = 0.05, odds 19:1 in favour of the hypothesis of a chance geometry" (p. 53). But assuming that the churches are deliberately aligned in some

[1] This is a revised and translated version of Jerkert (2002). Having read my Swedish article, Niels Lind made the following statement for Erling Haagensen's new book: "Mr Jerkert thinks that there are two flaws in my META article. First, mixing up two different hypotheses; those interested can carefully read the two contributions and see that it is Mr Jerkert who is confused. Second flaw: using prior probabilities that are too favourable to my conclusion. Indeed, if you choose in advance not to believe a hypothesis, then no analysis, Bayesian or otherwise, can shake that belief" (Haagensen 2003, pp. 135-6). Lind obviously failed to understand my Swedish arguments. In the present version, I have tried to make my case even clearer.

way, there are *a priori* thousands of hypotheses just as likely as the Christiansø hypothesis. There is a multitude of ways of locating churches in a non-random fashion. Even if we would accept the probability of chance geometry to be 95%, the probability of *c* is therefore not 5% but much lower. (The words *a priori* are important. I will get back to this.)

I should add, for clarity, that even if it would be reasonable to represent (3) by a chance model, it is not unobjectionable. In reality, churches are of course *never* randomly located. They are always built by thinking humans, the locations being determined by several factors such as the distance to other churches, demographic characteristics, the topography, and so forth (cf. Rabow 2000, especially p. 123).

NONSENSE ASSIGNMENT OF PRIOR PROBABILITY

The second flaw in Lind is more serious. As we have seen, Lind thinks that his choice of prior probability is "reasonably skeptical". He refers to the fact that 95% confidence is conventional in many natural, biological and social sciences. He also claims that a probability of about 90-95% should be labeled "almost certain", according to a study on words-to-probability mapping. These are Lind's only lines of reasoning in assessing the prior probability. His argument is not convincing.

Bayesian inference is, in essence, a weighing according to Bayes's theorem of some knowledge in advance (*a priori*) and some factual evidence, in order to reach a final probability estimate (*a posteriori*). The *a priori* assessment, reflecting background knowledge, may be more or less well-founded. For this reason, authors of technical papers frequently use vague, non-committal prior probabilities, so that each reader may adjust the calculations according to his or her knowledge in the field (which might be greater than the author's). However, nothing prevents an author from performing a deeper analysis; this is in any case what the reader has to do if (s)he is not satisfied with the author's assessments.

We have a problem like this: What is the prior probability that church-builders at Bornholm during medieval times deliberately located churches according to certain patterns? We are allowed to use knowledge of history, technology, geography, etc., but we are *not* allowed to take into account that four of the churches are *in fact* aligned with Christiansø, because this is the evidential part of what goes into the bayesian analysis. I suggest we divide the problem into smaller parts which are easier to tackle, eventually weighing all parts together. The smaller questions might be, for example: What is the topography of Bornholm like? What kind of technical equipment and knowledge would be required to locate the churches in patterns over certain distances? Was this technique available in Bornholm at this time? Would a deliberate alignment of churches be relevant in the worldviews of the era? If we specifically want to estimate the probability of the Christiansø hypothesis, we shall have to add a question like this: Was Christiansø known for its religious significance at this time?

According to Lind, these questions are "fuzzy" and cannot be part of a serious analysis (p. 50). While it is perfectly true that they are fuzzy, dealing with them happens to be the only way of assessing the prior probability in a responsible way. Fuzzy questions are regularly found at the very core of bayesian prior probability estimations. I am surprised to learn that Lind is unaware of this.

Let us chart the logical consequences of Lind's refusal to analyse fuzzy questions.

What is the probability that extraterrestrials built the Egyptian pyramids? Well, it depends on the evidence. As far as I know, there is no evidence for such a hypothesis, and historians face no great problems explaining the buildings in wholly terrestrial terms. I believe that a proper analysis would assign a very low probability to the extraterrestrial hypo-

thesis, far below 5%. Lind, on the other hand, would end up with a 5% estimate, since he does not want to get involved with fuzzy questions.

A second example: What is the likelihood that I am actually the son of the Swedish king Gustavus I (who died in 1560)? The answer is not absolutely zero. We could imagine a fantastic series of events including deep-frozen semen from the 16th century king, eventually leading to my birth. Anyone wishing to estimate this likelihood must assess the credibility of the circumstances required to make the hypothesis true. Again, for Lind a proper probability estimate is 5%. In fact, Lind's method attributes prior probabilities of 5% to *any* improbable hypothesis. This is, of course, absurd.

My second example can serve to clarify one of my earlier arguments, too. Even without analysing the likelihood of fantastic historical hypotheses, we can firmly say that the probability that I am the son of Gustavus I is lower than one in a million. This is so because there is no reason (as far as I know) to believe that the probability that *I* am the son of Gustavus I is greater than the probability that *someone else* presently living in Sweden is his son (or daughter). Thus, even if we were convinced that *there is* a person living today who is the child of Gustavus I, the probability of that person being *me* is no more than one in several millions, unless we can give some reasons for treating my case differently from all others.

Likewise, the probability that the churches of Bornholm were deliberately aligned *in any way* must be split on *all* conceivable configurations. One of the thousands of possible configurations is aligning four churches (out of 15) pairwise with Christiansø. The prior probability of the Christiansø hypothesis cannot exceed one in several thousands, unless we can give some special arguments for it. Lind has presented no such argument.

CONCLUSIONS

Lind mixed up two different hypotheses, and for this reason his calculations are questionable. More serious still, in assigning the prior probability Lind did not analyse the factual circumstances, but relied on verbal probability expressions. His line of reasoning implies that any improbable hypothesis could be attributed a prior probability of 5%, which is absurd. Lind's approach is un-bayesian. I conclude that his analysis is mistaken.

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